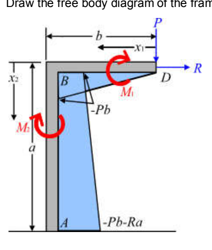
Advanced Strength and Applied Elasticity (4th Edition)

Chapter 10, Problem 1P

Step-by-step solution

Step 1 of 13

Draw the free body diagram of the frame.



Comment

Step 2 of 13

Apply the equations of equilibrium and calculate the bending moment in the horizontal member.

$$\sum M = 0$$
$$M_1 + Px_1 = 0$$
$$M_1 = -Px_1$$

Here, *P* is vertical load.

Apply the equations of equilibrium and determine the axial force of horizontal beam.

$$\sum F_x = 0$$
$$R - N_1 = 0$$
$$N_1 = R$$

Here, *R* is horizontal load.

Comments (2)

Step 3 of 13

Apply the equations of equilibrium and calculate the bending moment in vertical beam.

$$\sum M = 0$$
$$M_2 + P(b) + R(x_2) = 0$$
$$M_2 = -Pb - Rx_2$$

Apply the equations of equilibrium and calculate the vertical force.

$$\sum F_y = 0$$
$$N_2 - P = 0$$
$$N_2 = P$$

Determine the partial derivatives of M_1 with respect to the load R.

$$M_1 = -Px_1$$
$$\frac{\partial M_1}{\partial R} = 0$$

Comment

Step 4 of 13

Determine the partial derivatives of M_2 with respect to the load R.

$$M_2 = -Pb - Rx_2$$
$$\frac{\partial M_2}{\partial R} = -x_2$$

Determine the partial derivatives of N_1 with respect to the load R.

$$N_1 = R$$
$$\frac{\partial N_1}{\partial R} = 1$$

Determine the partial derivatives of N_2 with respect to the load R.

$$N_2 = P$$
$$\frac{\partial N_2}{\partial R} = 0$$

The horizontal deflection at *D* is assumed to go in the same direction as *R*.

Comment

Step 5 of 13

Apply the Castigliano's theorem and determine the horizontal deflection at *D*, δ_{Dh}.

$$\delta_{Dh} = \left[\begin{aligned} &\frac{1}{EI}\int_0^b M_1\left(\frac{\partial M_1}{\partial R}\right)dx + \frac{1}{EA}\int_0^b N_1\left(\frac{\partial N_1}{\partial R}\right)dx \\ &+ \frac{1}{EI}\int_0^a M_2\left(\frac{\partial M_2}{\partial R}\right)dx + \frac{1}{EA}\int_0^a N_2\left(\frac{\partial N_2}{\partial R}\right)dx \end{aligned}\right]$$

Substitute 0 for $\frac{\partial M_1}{\partial R}$, $-x_2$ for $\frac{\partial M_2}{\partial R}$, 1 for $\frac{\partial N_1}{\partial R}$, 0 for $\frac{\partial N_2}{\partial R}$, $(-Pb - Rx_2)$ for M_2, R for N_1, and P for N_2.

$$\delta_{Dh} = \left[\begin{aligned} &\frac{1}{EI}\int_0^b M_1(0)dx + \frac{1}{EA}\int_0^b (R)(1)dx \\ &+ \frac{1}{EI}\int_0^a (-Pb - Rx_2)(-x_2)dx + \frac{1}{EA}\int_0^a N_2(0)dx \end{aligned}\right]$$
$$= \frac{R}{EA}(x)_0^b + \frac{1}{EI}\int_0^a \left(Pbx_2 Rx_2^2\right)dx$$
$$= \frac{Rb}{EA} + \frac{Pb}{EI}\left(\frac{x_2^2}{2}\right)_0^a + \frac{R}{EI}\left(\frac{x_2^3}{3}\right)_0^a$$
$$= \left(\frac{Rb}{EA} + \frac{Pba^2}{2EI} + \frac{Ra^3}{3EI}\right)(\rightarrow)$$

Therefore, the horizontal deflection δ_{Dh} is $\boxed{\frac{Rb}{EA} + \frac{Pba^2}{2EI} + \frac{Ra^3}{3EI}(\rightarrow)}$.

Comment

Step 6 of 13

Determine the partial derivatives of M_1 with respect to P.

$$M_1 = -Px_1$$
$$\frac{\partial M_1}{\partial P} = -x_1$$

Calculate the partial derivatives of M_2 with respect to P.

$$M_2 = -Pb - Rx_2$$
$$\frac{\partial M_2}{\partial P} = -b$$

Calculate the partial derivatives of N_1 with respect to P.

$$N_1 = R$$
$$\frac{\partial N_1}{\partial P} = 0$$

Calculate the partial derivatives of N_2 with respect to P.

$$N_2 = P$$
$$\frac{\partial N_2}{\partial P} = 1$$

Comment

Step 7 of 13

Use Castigliano's theorem to determine the vertical deflection at *D*, δ_{Dv}.

$$\delta_{Dv} = \left[\begin{aligned} &\frac{1}{EI}\int_0^b M_1\left(\frac{\partial M_1}{\partial P}\right)dx + \frac{1}{EA}\int_0^b N_1\left(\frac{\partial N_1}{\partial P}\right)dx \\ &+ \frac{1}{EI}\int_0^a M_2\left(\frac{\partial M_2}{\partial P}\right)dx + \frac{1}{EA}\int_0^a N_2\left(\frac{\partial N_2}{\partial P}\right)dx \end{aligned}\right]$$

Substitute $-x_1$ for $\frac{\partial M_1}{\partial P}$, $-b$ for $\frac{\partial M_2}{\partial P}$, 0 for $\frac{\partial N_1}{\partial P}$, 1 for $\frac{\partial N_2}{\partial P}$, $-Px_1$ for M_1, $(-Pb - Rx_2)$ for M_2, R for N_1, and P for N_2.

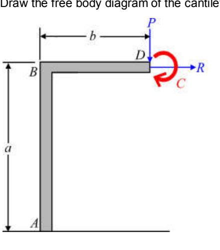
$$\delta_{Dv} = \left[\begin{aligned} &\frac{1}{EI}\int_0^b (-Px_1)(-x_1)dx + \frac{1}{EA}\int_0^b N_1(0)dx \\ &+ \frac{1}{EI}\int_0^a (-Pb - Rx_2)(-b)dx + \frac{1}{EA}\int_0^a (P)(1)dx \end{aligned}\right]$$
$$= \frac{1}{EI}\int_0^b Px_1^2 dx + \frac{1}{EI}\int_0^a \left(Pb^2 + Rx_2 b\right)dx + \frac{1}{EA}\int_0^a P dx$$
$$= \frac{1}{EI}\left(\frac{x_1^3}{3}\right)_0^b + \frac{1}{EI}\left(Pb^2 + R\frac{x_2^2}{2}b\right)_0^a + \frac{1}{EA}P\left(\frac{x^2}{2}\right)_0^a$$
$$= \frac{Pb^3}{3EI} + \frac{Pb^2a}{EI} + \frac{Ra^2b}{2EI} + \frac{Pa}{EA}(\downarrow)$$

Since, the answer is positive, the deflection is in the downward direction of the force *P*, as was assumed.

Comment

Step 8 of 13

Draw the free body diagram of the cantilever beam with fictitious moment C.



Comment

Step 9 of 13

Apply the equations of equilibrium and calculate new bending moment for the horizontal beam by taking *C* into account in addition to forces *P* and *R*.

$$\sum M = 0$$
$$M_1 + Px_1 + C = 0$$
$$M_1 = -Px_1 - C$$

Since there is no actual moment applied at point *D*, a fictitious moment applied at point *D*, and it is called *C*. The value of *C* is zero.

Apply the equations of equilibrium and calculate the horizontal forces.

$$\sum F_x = 0$$
$$N_1 - R = 0$$
$$N_1 = R$$

Comment

Step 10 of 13

Apply the equations of equilibrium and calculate the moments in the vertical beam.

$$\sum M = 0$$
$$M_2 + Pb + Rx_2 + C = 0$$
$$M_2 = -Pb - Rx_2 - C$$

Apply the equations of equilibrium and calculate the vertical forces.

$$\sum F_y = 0$$
$$N_2 - P = 0$$
$$N_2 = P$$

Comment

Step 11 of 13

Calculate the partial derivative of M_1 with respect to C.

$$M_1 = -Px_1 - C$$
$$\frac{dM_1}{dC} = \frac{d}{dC}(-Px_1 - C)$$
$$\frac{dM_1}{dC} = -1$$

Calculate the partial derivative of M_2 with respect to C.

$$M_2 = -Pb - Rx_2 - C$$
$$\frac{dM_2}{dC} = \frac{d}{dC}(-Pb - Rx_2 - C)$$
$$\frac{dM_2}{dC} = -1$$

Comment

Step 12 of 13

Calculate the partial derivative of N_1 with respect to C.

$$N_1 = R$$
$$\frac{dN_1}{dC} = \frac{d}{dC}(R)$$
$$\frac{dN_1}{dC} = 0$$

Calculate the partial derivative of N_2 with respect to C.

$$N_2 = P$$
$$\frac{dN_2}{dC} = \frac{dP}{dC}$$
$$\frac{dN_2}{dC} = 0$$

Comment

Step 13 of 13

Use Castigliano's theorem to determine the rotation at *D*, θ_D.

$$\theta_D = \left[\begin{aligned} &\frac{1}{EI}\int_0^b M_1\left(\frac{\partial M_1}{\partial C}\right)dx + \frac{1}{EA}\int_0^b N_1\left(\frac{\partial N_1}{\partial C}\right)dx \\ &+ \frac{1}{EI}\int_0^a M_2\left(\frac{\partial M_2}{\partial C}\right)dx + \frac{1}{EA}\int_0^a N_2\left(\frac{\partial N_2}{\partial C}\right)dx \end{aligned}\right]$$

Substitute $(-Px_1 - C)$ for M_1, R for N_1, $(-Pb - Rx_2 - C)$ for M_2, -1 for $\frac{\partial M_1}{\partial C}$, -1 for $\frac{\partial M_2}{\partial C}$, 0 for $\frac{\partial N_1}{\partial C}$, 0 for $\frac{\partial N_2}{\partial C}$, 0 for *C*

$$\theta_D = \frac{1}{EI}\int_0^b (Px_1 + C)dx + \frac{1}{EI}\int_0^a (Pb + Rx_2 + C)dx$$
$$= \frac{1}{EI}P\left(\frac{x_1^2}{2}\right)_0^b + \frac{1}{EI}\left[Pb(x)_0^a + R\left(\frac{x_2^2}{2}\right)_0^a\right]$$
$$= \left(\frac{Pb^2}{2EI}\right) + \left[\frac{Pb(a)}{EI} + \frac{Ra^2}{2EI}\right]$$

Therefore, the rotation at *D*, θ_D is $\boxed{\frac{Pb^2}{2EI} + \frac{Pba}{EI} + \frac{Ra^2}{2EI}(\text{C.W.})}$

Since, the answer is positive, the rotation is in the clockwise direction of the fictitious moment *C*, as assumed.

Comment