

Advanced Strength and Applied Elasticity (4th Edition)

Chapter 10, Problem 1P

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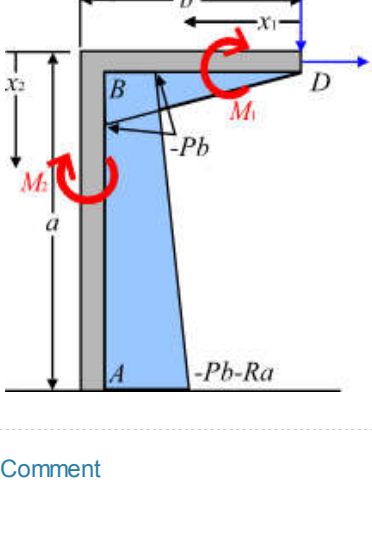
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Step-by-step solution

Step 1 of 13

Draw the free body diagram of the frame.



Comment

Step 2 of 13

Apply the equations of equilibrium and calculate the bending moment in the horizontal member.

$$\sum M = 0$$
$$M_1 + Px_1 = 0$$
$$M_1 = -Px_1$$

Here, P is vertical load.

Apply the equations of equilibrium and determine the axial force of horizontal beam.

$$\sum F_x = 0$$
$$R - N_1 = 0$$
$$N_1 = R$$

Here, R is horizontal load.

Comments (2)

Step 3 of 13

Apply the equations of equilibrium and calculate the bending moment in vertical beam.

$$\sum M = 0$$
$$M_2 + P(b) + R(x_2) = 0$$
$$M_2 = -Pb - Rx_2$$

Apply the equations of equilibrium and calculate the vertical force.

$$\sum F_y = 0$$
$$N_2 - P = 0$$
$$N_2 = P$$

Determine the partial derivatives of M_1 with respect to the load R .

$$M_1 = -Px_1$$
$$\frac{\partial M_1}{\partial R} = 0$$

Comment

Step 4 of 13

Determine the partial derivatives of M_2 with respect to the load R .

$$M_2 = -Pb - Rx_2$$
$$\frac{\partial M_2}{\partial R} = -x_2$$

Determine the partial derivatives of N_1 with respect to the load R .

$$N_1 = R$$
$$\frac{\partial N_1}{\partial R} = 1$$

Determine the partial derivatives of N_2 with respect to the load R .

$$N_2 = P$$
$$\frac{\partial N_2}{\partial R} = 0$$

The horizontal deflection at D is assumed to go in the same direction as R .

Comment

Step 5 of 13

Apply the Castiglano's theorem and determine the horizontal deflection at D , δ_{DH} .

$$\delta_{DH} = \left[\frac{1}{EI} \int_0^b M_1 \left(\frac{\partial M_1}{\partial R} \right) dx + \frac{1}{EA} \int_0^a N_1 \left(\frac{\partial N_1}{\partial R} \right) dx \right] + \left[\frac{1}{EI} \int_0^b M_2 \left(\frac{\partial M_2}{\partial R} \right) dx + \frac{1}{EA} \int_0^a N_2 \left(\frac{\partial N_2}{\partial R} \right) dx \right]$$

Substitute 0 for $\frac{\partial M_1}{\partial R}$, $-x_2$ for $\frac{\partial M_2}{\partial R}$, 1 for $\frac{\partial N_1}{\partial R}$, 0 for $\frac{\partial N_2}{\partial R}$, $(-Pb - Rx_2)$ for M_2 , R for N_1 , and P for N_2 .

$$\delta_{DH} = \left[\frac{1}{EI} \int_0^b M_1(0) dx + \frac{1}{EA} \int_0^a (R)(1) dx \right] + \left[\frac{1}{EI} \int_0^b (-Pb - Rx_2)(-x_2) dx + \frac{1}{EA} \int_0^a (P)(1) dx \right]$$
$$= \frac{R}{EA} (x_1)^3 + \frac{1}{EI} \int_0^b (Phx_2Rx_2^2) dx$$
$$= \frac{Rb}{EA} + \frac{Ph}{EI} \left(\frac{x_2^3}{3} \right)_0^b + \frac{R}{EI} \left(\frac{x_2^2}{2} \right)_0^b$$
$$= \left(\frac{Rb}{EA} + \frac{Pha^2}{2EI} + \frac{Ra^2}{3EI} \right) (-\rightarrow)$$

Therefore, the horizontal deflection δ_{DH} is $\left(\frac{Rb}{EA} + \frac{Pha^2}{2EI} + \frac{Ra^2}{3EI} \right) (-\rightarrow)$.

Comment

Step 6 of 13

Determine the partial derivatives of M_1 with respect to P .

$$M_1 = -Px_1$$
$$\frac{\partial M_1}{\partial P} = -x_1$$

Calculate the partial derivatives of M_2 with respect to P .

$$M_2 = -Pb - Rx_2$$
$$\frac{\partial M_2}{\partial P} = -b$$

Calculate the partial derivatives of N_1 with respect to P .

$$N_1 = R$$
$$\frac{\partial N_1}{\partial P} = 0$$

Calculate the partial derivatives of N_2 with respect to P .

$$N_2 = P$$
$$\frac{\partial N_2}{\partial P} = 1$$

Comment

Step 7 of 13

Use Castiglano's theorem to determine the vertical deflection at D , δ_{DV} .

$$\delta_{DV} = \left[\frac{1}{EI} \int_0^b M_1 \left(\frac{\partial M_1}{\partial P} \right) dx + \frac{1}{EA} \int_0^a N_1 \left(\frac{\partial N_1}{\partial P} \right) dx \right] + \left[\frac{1}{EI} \int_0^b M_2 \left(\frac{\partial M_2}{\partial P} \right) dx + \frac{1}{EA} \int_0^a N_2 \left(\frac{\partial N_2}{\partial P} \right) dx \right]$$

Substitute $-x_1$ for $\frac{\partial M_1}{\partial P}$, $-b$ for $\frac{\partial M_2}{\partial P}$, 0 for $\frac{\partial N_1}{\partial P}$, 1 for $\frac{\partial N_2}{\partial P}$, $-Px_1$ for M_1 , $(-Pb - Rx_2)$ for M_2 , R for N_1 , and P for N_2 .

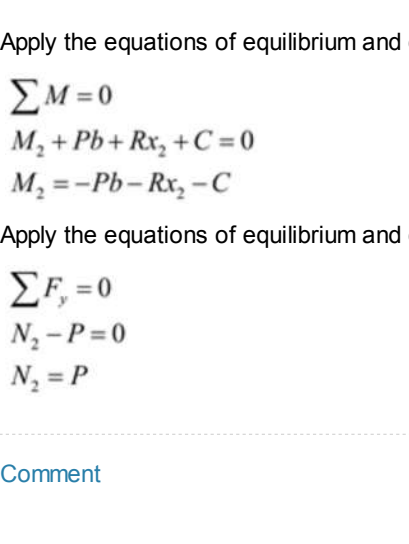
$$\delta_{DV} = \left[\frac{1}{EI} \int_0^b (-Px_1)(-x_1) dx + \frac{1}{EA} \int_0^a (R)(1) dx \right] + \left[\frac{1}{EI} \int_0^b (-Pb - Rx_2)(-b) dx + \frac{1}{EA} \int_0^a (P)(1) dx \right]$$
$$= \frac{1}{EI} \int_0^b Px_1^2 dx + \frac{1}{EI} \int_0^b (Ph^2 + Rx_2b) dx + \frac{1}{EA} \int_0^a P dx$$
$$= \frac{1}{EI} \left(\frac{x_1^3}{3} \right)_0^b + \frac{1}{EI} \left(Phb^2 + R \frac{b^2}{2} \right)_0^b + \frac{1}{EA} \left(\frac{x_2^2}{2} \right)_0^a$$
$$= \frac{Pb^3}{3EI} + \frac{Ph^2a}{EI} + \frac{Ra^2b}{2EI} + \frac{Pa}{EA} \left(\frac{1}{2} \right)$$

Since, the answer is positive, the deflection is in the downward direction of the force P , as was assumed.

Comment

Step 8 of 13

Draw the free body diagram of the cantilever beam with fictitious moment C .



Comment

Step 9 of 13

Apply the equations of equilibrium and calculate new bending moment for the horizontal beam by taking C into account in addition to forces P and R .

$$\sum M = 0$$
$$M_1 + Px_1 + C = 0$$
$$M_1 = -Px_1 - C$$

Since there is no actual moment applied at point D , a fictitious moment applied at point D , and it is called C . The value of C is zero.

Apply the equations of equilibrium and calculate the horizontal forces.

$$\sum F_x = 0$$
$$N_1 - R = 0$$
$$N_1 = R$$

Comment

Step 10 of 13

Apply the equations of equilibrium and calculate the moments in the vertical beam.

$$\sum M = 0$$
$$M_2 + Pb + Rx_2 + C = 0$$
$$M_2 = -Pb - Rx_2 - C$$

Apply the equations of equilibrium and calculate the vertical forces.

$$\sum F_y = 0$$
$$N_2 - P = 0$$
$$N_2 = P$$

Comment

Step 11 of 13

Calculate the partial derivative of M_1 with respect to C .

$$M_1 = -Px_1 - C$$
$$\frac{dM_1}{dC} = \frac{d}{dC} (-Px_1 - C)$$
$$\frac{dM_1}{dC} = -1$$

Calculate the partial derivative of M_2 with respect to C .

$$M_2 = -Pb - Rx_2 - C$$
$$\frac{dM_2}{dC} = \frac{d}{dC} (-Pb - Rx_2 - C)$$
$$\frac{dM_2}{dC} = -1$$

Comment

Step 12 of 13

Calculate the partial derivative of N_1 with respect to C .

$$N_1 = R$$
$$\frac{dN_1}{dC} = \frac{d}{dC} (R)$$
$$\frac{dN_1}{dC} = 0$$

Calculate the partial derivative of N_2 with respect to C .

$$N_2 = P$$
$$\frac{dN_2}{dC} = \frac{dP}{dC}$$
$$\frac{dN_2}{dC} = 0$$

Comment

Step 13 of 13

Use Castiglano's theorem to determine the rotation at D , θ_D .

$$\theta_D = \left[\frac{1}{EI} \int_0^b M_1 \left(\frac{\partial M_1}{\partial C} \right) dx + \frac{1}{EA} \int_0^a N_1 \left(\frac{\partial N_1}{\partial C} \right) dx \right] + \left[\frac{1}{EI} \int_0^b M_2 \left(\frac{\partial M_2}{\partial C} \right) dx + \frac{1}{EA} \int_0^a N_2 \left(\frac{\partial N_2}{\partial C} \right) dx \right]$$

Substitute $(-Px_1 - C)$ for M_1 , R for N_1 , $(-Pb - Rx_2 - C)$ for M_2 , -1 for $\frac{\partial M_1}{\partial C}$, -1 for $\frac{\partial M_2}{\partial C}$, 0 for $\frac{\partial N_1}{\partial C}$, 0 for $\frac{\partial N_2}{\partial C}$.

$$\theta_D = \frac{1}{EI} \int_0^b (-Px_1 - C) dx + \frac{1}{EI} \int_0^b (-Pb - Rx_2 - C) dx$$
$$= \frac{1}{EI} P \left(\frac{x_1^2}{2} \right)_0^b + \frac{1}{EI} \left[Pb(x_1)^2 + R \left(\frac{x_2^2}{2} \right)_0^b \right]$$
$$= \left(\frac{Pb^2}{2EI} \right) + \left[\frac{Pb(a)}{EI} + \frac{Ra^2}{2EI} \right]$$

Therefore, the rotation at D , θ_D is $\left[\frac{Pb^2}{2EI} + \frac{Pba}{EI} + \frac{Ra^2}{2EI} \right] (C.W.)$

Since, the answer is positive, the rotation is in the clockwise direction of the fictitious moment C , as assumed.

Comment

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